Modia - A Domain Specific Extension of Julia for Modeling and Simulation

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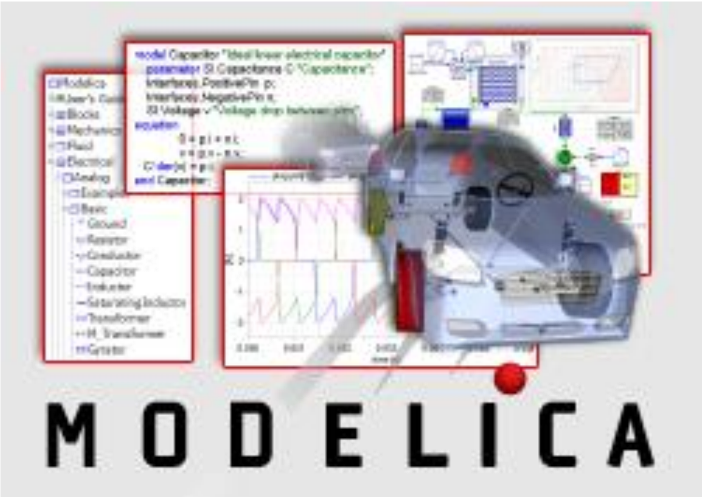
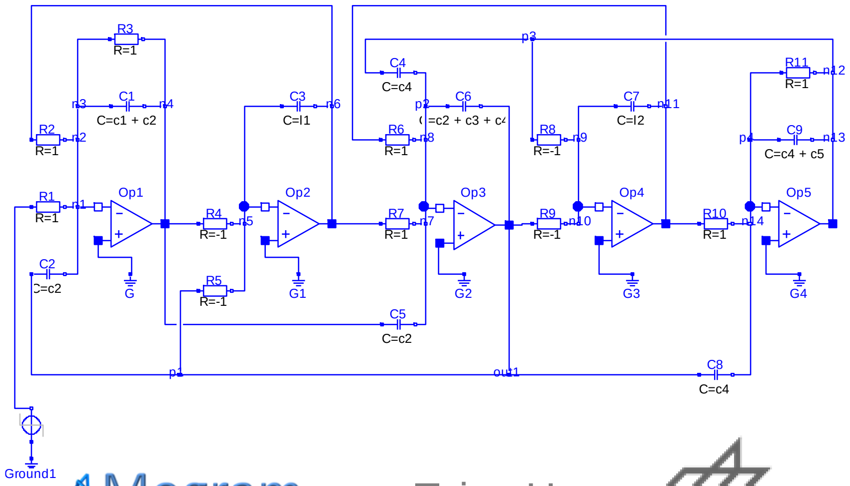
Outline

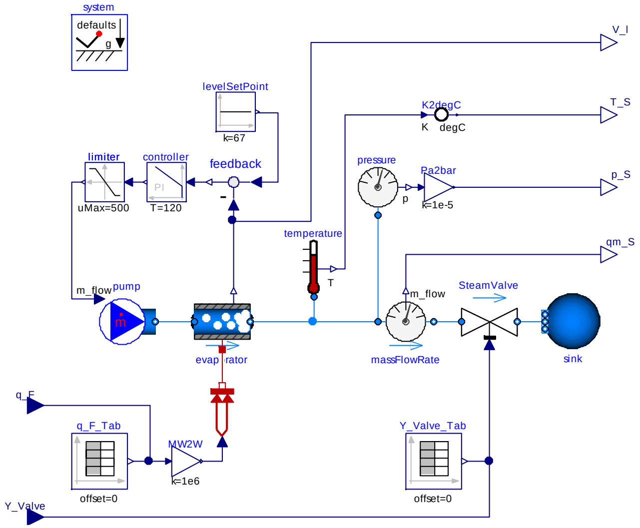
- Rationale for *Modia* project
- *Modia* Language by examples
- *Modia* Prototype
- Summary

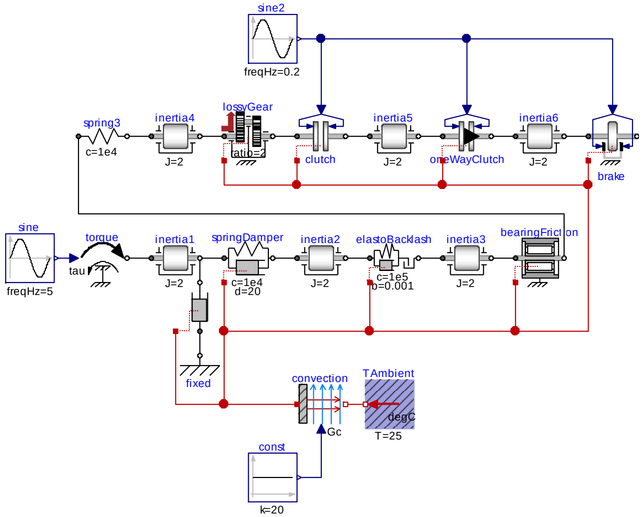
Modelica for Systems Modeling

- www.Modelica.org
- A formal language to capture modeling knowhow
- Equation based language - for convenience
- Object oriented - for reuse
- System topology - by connections
- Terminal definitions - connectors
- Icons AND equations - not only symbols









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Modelica Basics

- Object- and equation-oriented modeling language
- Successfully utilized in industry for modeling, simulating and optimizing complex systems such as automobiles, aircraft, power systems, etc.
- The dynamic behavior of system components is modelled by equations, for example, mass- and energy-balances.
  - Ordinary Differential Equations
  - Algebraic Equations
  - = DAE (Differential Algebraic Equations)
- Modelica is quite different from ordinary programming languages since equations with mathematical expressions on both sides of the equals sign are allowed.
- Structural and symbolic methods are used to compile such equations into efficient executable code.

Why Modia?

- New needs of modeling features are requested
- Need an experimental language platform

- Modelica specification is becoming large and hard to comprehend
- Could be complemented by a reference implementation

- Functions/Algorithms in Modelica are not powerful
  - no advanced data structures such as union types, no matching construct, no type inference, etc
- Possibility to utilize other language efforts for functions
- Julia has perfect scientific computing focus
- Modia - Julia macro set

We hope to use this work to make contributions to the Modelica effort

Modia – “Hello Physical World” model

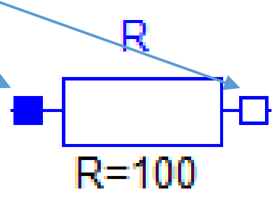
```
@model FirstOrder begin
  x = Variable(start=1)
  T = Parameter(0.5, info="Time constant")
  u = 2.0 # Same as Parameter(2.0)
@equations begin
  T*der(x) + x = u
  end
end
```

Modelica

```
model FirstOrder
  Real x(start=1);
  parameter Real T=0.5 "Time constant";
  parameter Real u = 2.0;
equation
  T*der(x) + x = u;
end M;
```

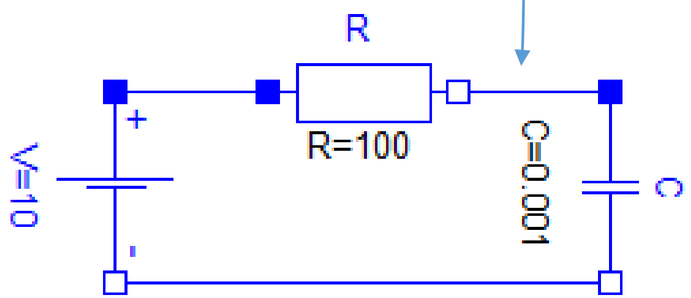
Connectors and Components - Electrical

```
@model Pin begin
 v=Float()
 i=Float(flow=true)
end

@model OnePort begin
 p=Pin()
 n=Pin()
 v=Float()
 i=Float()
@equations begin
 v = p.v - n.v # Voltage drop
 0 = p.i + n.i # KCL within component
 i = p.i
 end
end

@model Resistor begin # Ideal linear electrical resistor
 @extends OnePort()
 @inherits i, v
 R=1 # Resistance
@equations begin
 R*i = v
 end
end
```



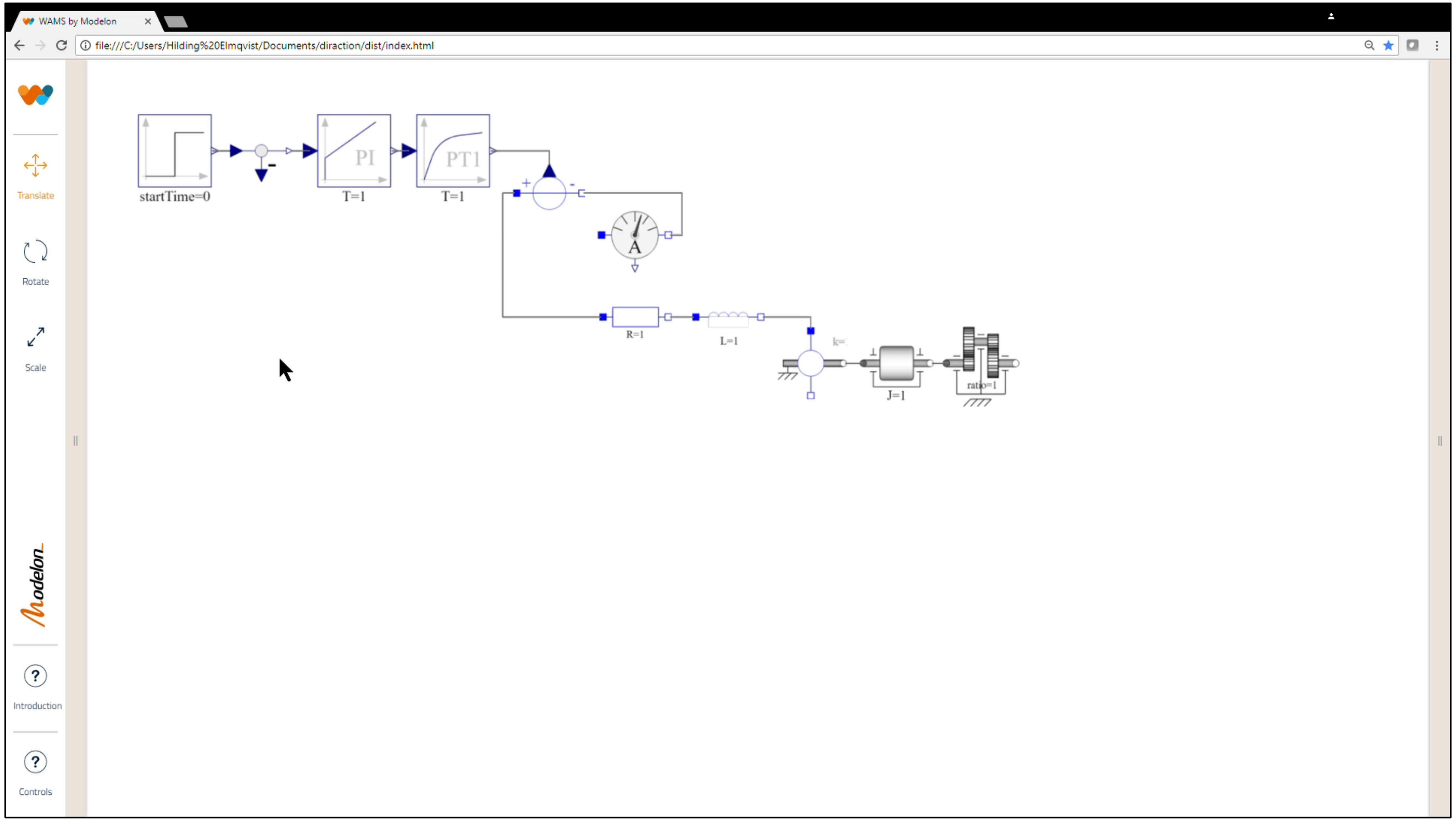
Coupled Models - Electrical Circuit

```
@model LPfilter begin
  R = Resistor(R=100)
  C = Capacitor(C=0.001)
  V = ConstantVoltage(V=10)
@equations begin
  connect(R.n, C.p)
  connect(R.p, V.p)
  connect(V.n, C.n)
  end
end
```

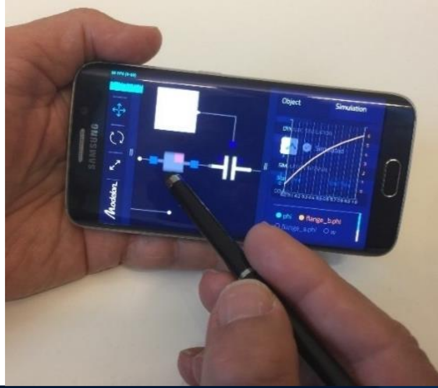


Web App – for connecting components

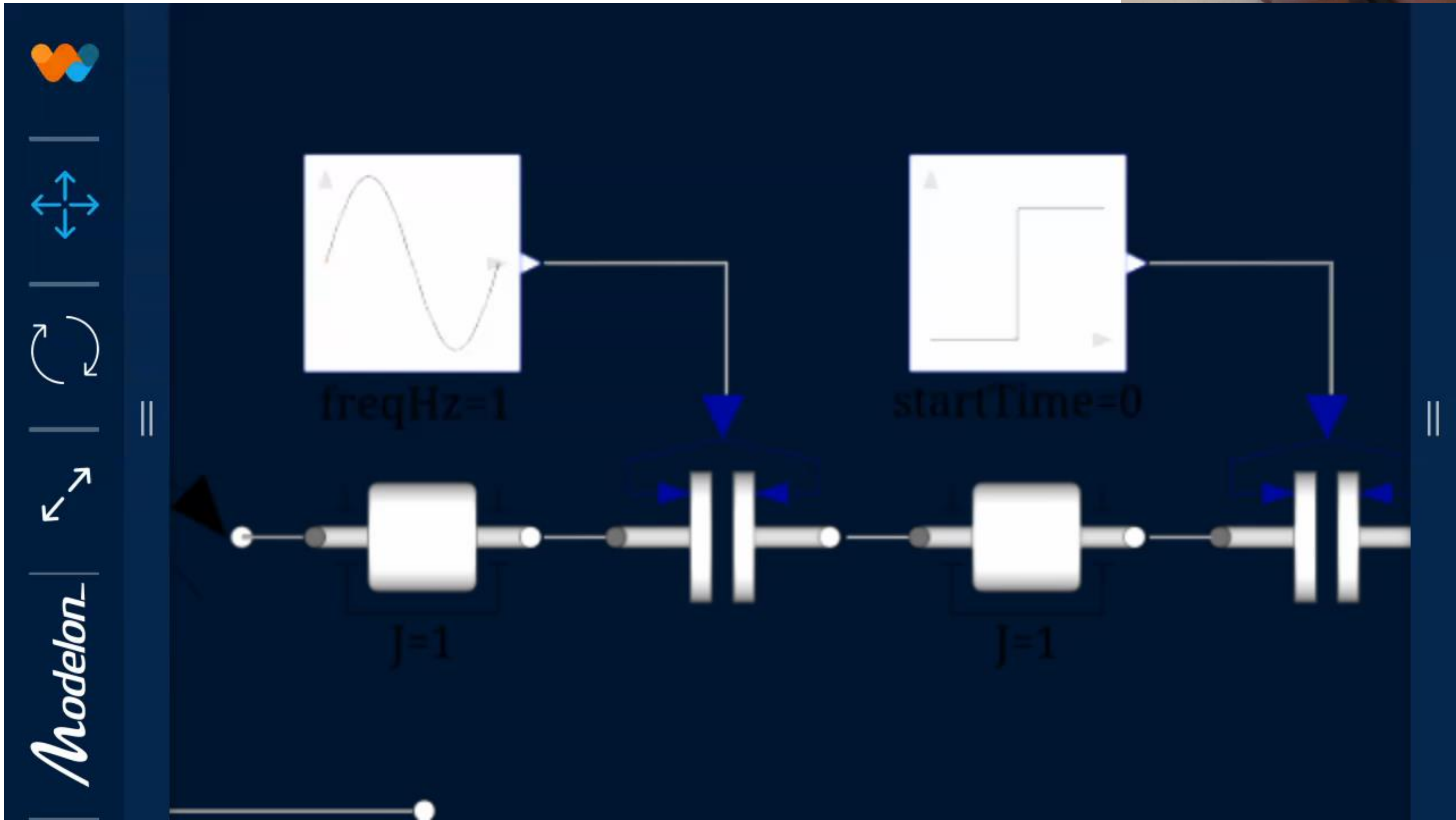
- Bachelor thesis project
- Create, connect, set parameters, simulate, plot, animate in 3D
- Automatic placement and routing
- Together with Modelon AB



Web App – on smart phone



- Scenario:
- Working in your autonomous car



Variable Constructor

In general: time varying variable with attributes:

```
type Variable
   variability::Variability
   T::DataType
   size
   value
   unit::SIUnits.SIUnit
   min
   max
   start
   nominal
   info::AbstractString
   flow::Bool
end
```

Short version:

```
Var(; args...) = Variable(; args...)
```

Specialization for parameters:

```
Parameter(value; args...) = Var(variability=parameter, value=value; args...)
```

Variable Declarations

```
# With Float64 type
v1 = Var(T=Float64)

# With array type
array = Var(T=Array{Float64,1})
matrix = Var(T=Array{Float64,2})

# With fixed array sizes
scalar = Var(T=Float64, size=())
array3 = Var(T=Float64, size=(3,))
matrix3x3 = Var(T=Float64, size=(3,3))
```

```
# With unit
v2 = Var(T=Volt)

# Parameter with unit
m = 2.5kg
length = 5m
```

- Often natural to provide type and size information
- Unit handling with SIUnits.jl

Type Declarations

```
# Type declarations
Float3(; args...) = Var(T=Float64, size=(3,); args...)
Voltage(; args...) = Var(T=Volt; args...)

# Use of type declarations
v3 = Float3(start=zeros(3))
v4 = Voltage(size=(3,), start=[220.0, 220.0, 220.0]Volt)

Position(; args...) = Var(T=Meter; size=(), args...)

Position3(; args...) = Position(size=(3,); args...)
Rotation3(; args...) = Var(T=SIPrefix; size=(3,3), property=rotationGroup3D, args...)
```

- Reuse of type and size definitions
- Rotation matrices
  - Needed to handle closed kinematic loops

- Matrix equations
- DAE index reduction needed
- R_Rel equation differentiated (only phi time varying)
- Rotation3() implies ”special orthogonal group”, SO(3)

MultiBody modeling

```
@model Frame begin
  r_0 = Position3()
  R = Rotation3()
  f = Force3(flow=true) # Cut-force resolved in connector frame
  t = Torque3(flow=true) # Cut-torque resolved in connector frame
end

@model Revolute begin # Revolute joint (1 rotational degree-of-freedom, 2 potential states, optional axis flange)
  n = [0,0,1] # Axis of rotation resolved in frame_a

  frame_a = Frame()
  frame_b = Frame()

  phi = Angle(start=0)
  w = AngularVelocity(start=0)
  a = AngularAcceleration()
  tau = Torque()  # Driving torque in direction of axis of rotation
  R_rel =  Rotation3()

@equations begin
  R_rel = n*n' + (eye(3) - n*n')*cos(phi) –  skew(n)*sin(phi)

  w = der(phi)
  a = der(w)

  # relationships between quantities of frame_a and of frame_b
  frame_b.r_0 = frame_a.r_0
  frame_b.R = R_rel*frame_a.R
  frame_a.f + R_rel'*frame_b.f = zeros(3)
  frame_a.t + R_rel'*frame_b.t = zeros(3)

  # d'Alemberts principle
  tau = -n'*frame_b.t
  tau = 0 # Not driven
  end
end
```

Type and Size Inference - Generic switch

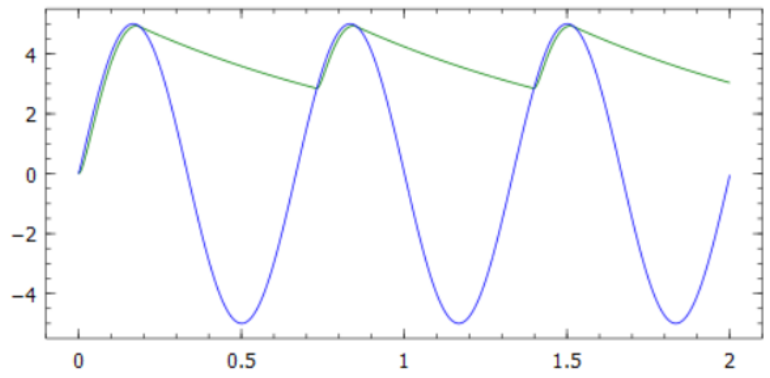
```
@model Switch begin
  sw=Boolean()
  u1=Variable()
  u2=Variable()
  y=Variable()
@equations begin
  y = if sw; u1 else u2 end
  end
end
```

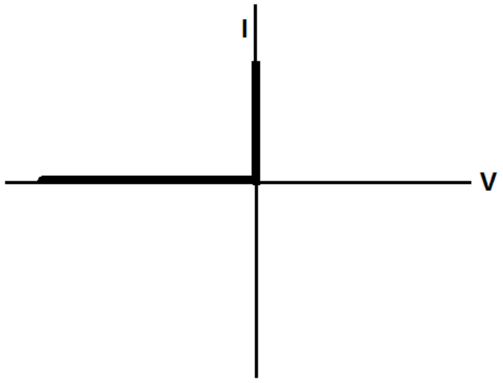
- Avoid duplication of models with different types
- Types and sizes can be inferred from the environment of a model or start values provided,
  either initial conditions for states or
  approximate start values for algebraic constraints.
- Inputs u1 and u2 and output y can be of any type

Discontinuities - State Events

- positive() and negative() introduces crossing functions

```
@model IdealDiode begin
  @extends OnePort()
  @inherits v, i
  s = Float(start=0.0)
@equations begin
  v = if positive(s); 0 else s end
  i = if positive(s); s else 0 end
  end
end
```

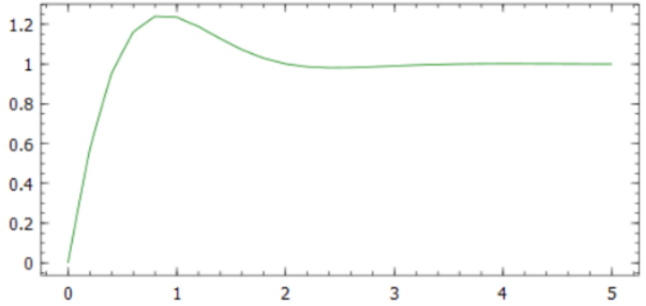


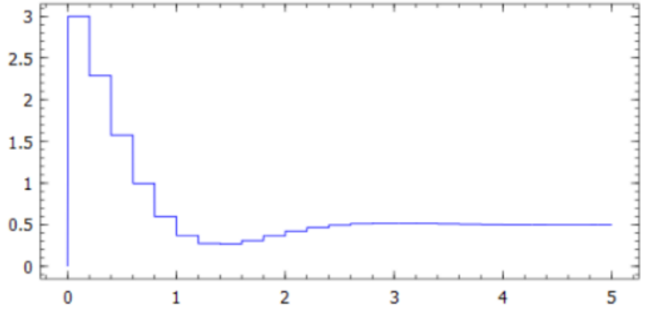


- Clock partitioning of equations
- Clock inference
- Clocked equations active at ticks

Synchronous Controllers

```
@model DiscretePIController begin
  K=1 # Gain
  Ti=1E10 # Integral time
  dt=0.1 # sampling interval
  ref=1 # set point
  u=Float(); ud=Float()
  y=Float(); yd=Float()
  e=Float(); i=Float(start=0)
@equations begin
  # sensor:
  ud = sample(u, Clock(dt))
  # PI controller:
  e = ref-ud
  i = previous(i, Clock(dt)) + e
  yd = K*(e + i/Ti)
  # actuator:
  y = hold(yd)
  end
end
```





Redeclaration of submodels

```
MotorModels = [Motor100KW, Motor200KW, Motor250KW] # array of Modia models
selectedMotor = motorConfig(  ) # Int

@model HybridCar begin
  @extends BaseHybridCar(
    motor = MotorModels[selectedMotor](),
    gear  = if gearOption1; Gear1(i=4) else Gear2(i=5) end)
end
```

- More powerful than **replaceable** in Modelica

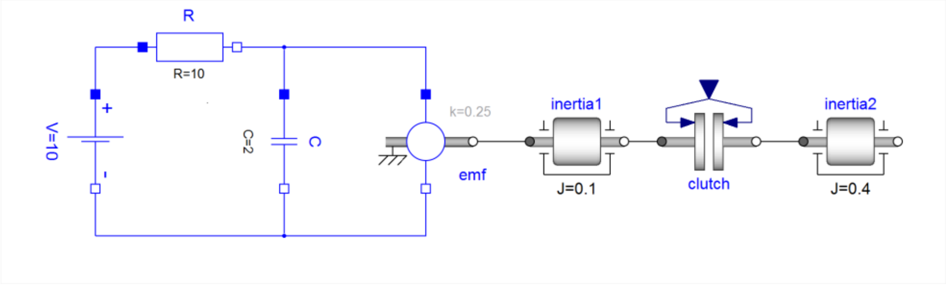
Indexing

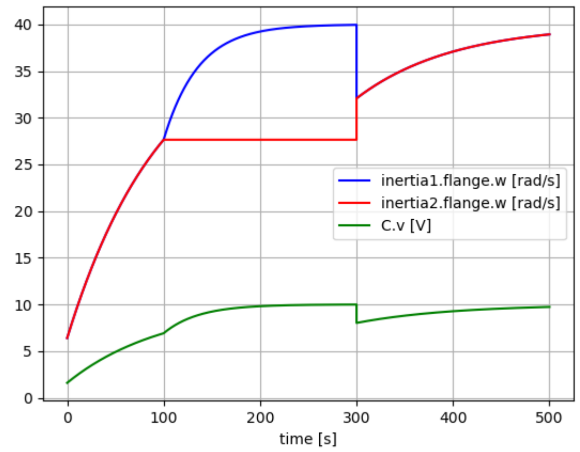
Conditional selection

Multi-mode Modeling

```
@model Clutch begin
 flange1 = Flange()
 flange2 = Flange()
 engaged = Boolean()
@equations begin
  if ! engaged
    flange1.tau = 0
    flange2.tau = 0
  else
    flange1.w = flange2.w
    flange1.tau + flange2.tau = 0
  end
 end
end
```

- Set of model equations and the **DAE index is changing** when clutch is engaged or disengaged
- New symbolic transformations and **just-in-time compilation** is made for each mode of the system
- Final results of variables before an event is used as initial conditions after the event
- Mode changes with conditional equations might introduces inconsistent initial conditions causing **Dirac impulses** to occur





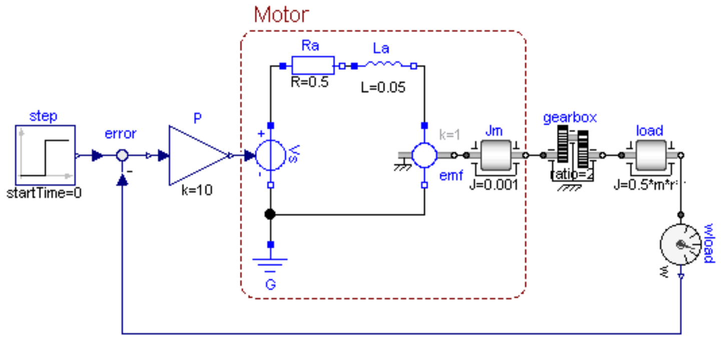
Functions and data structures

- built-in operator allInstances(v) creates a vector of all the variables v within all instances of the class where v is declared

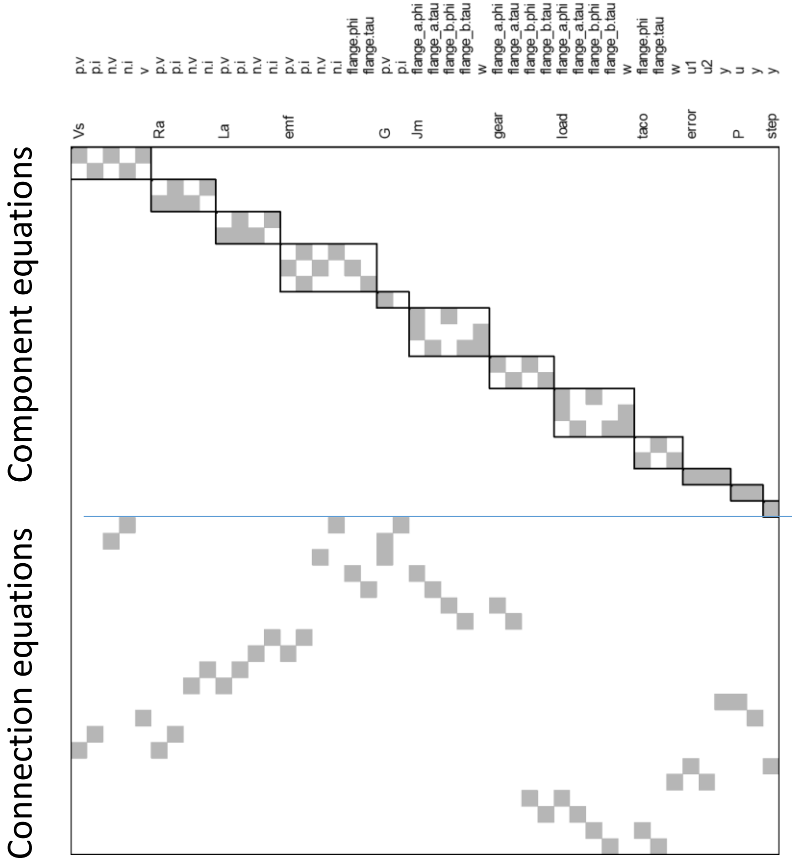
```
@model Ball begin
  r = Var()
  v = Var()
  f = Var()
  m = 1.0
@equations begin
  der(r) = v
  m*der(v) = f
  f = getForce(r, v, allInstances(r), allInstances(v), (r,v) -> (k*r + d*v))
  end
end

@model Balls begin
  b1 = Ball(r = Var(start=[0.0,2]), v = Var(start=[1,0]))
  b2 = Ball(r = Var(start=[0.5,2]), v = Var(start=[-1,0]))
  b3 = Ball(r = Var(start=[1.0,2]), v = Var(start=[0,0]))
end
```

```
function getForce(r, v, positions, velocities, contactLaw)
  force = zeros(2)
  for i in 1:length(positions)
    pos = positions[i]
    vel = velocities[i]
    if r != pos
      delta = r - pos
      deltaV = v - vel
      f = if norm(delta) < 2*radius;
        -contactLaw((norm(delta)-2*radius)*delta/norm(delta), deltaV)
        else zeros(2) end
      force += f
    end
  end
  return force
end
```

How To Simulate a Model



- Instantiate model, i.e. create sets of variables and equations
- Structurally analyze the equations
  - Which variable appear in which equation
  - Handle constraints (index reduction)
    - Differentiate certain equations
  - Sort the equations into execution order (BLT)
- Symbolically solve equations for unknowns and derivatives
- Generate code
- Numerically solve DAE
- Etc.



- Gives a sequence of subproblems
- Symbolically solve for variable in bold

BLT (Block Lower Triangular) form

error.u1 = step.offset+(if time < step.startTime then 0 else step.height)

error.y = error.u1-load.w

Vs.p.v = P.k*error.y

Ra.R*La.p.i = Vs.p.v-**Ra.n.v**

Jm.w = gear.ratio*load.w

emf.k*Jm.w = **La.n.v**

La.L***der(La.p.i)** = Ra.n.v-La.n.v

emf.flange.tau = -emf.k*La.p.i

// System of 4 simultaneous equations

der(Jm.w) = gear.ratio***der(load.w)**

Jm.J*der(Jm.w) = **Jm.flange_b.tau**-emf.flange.tau

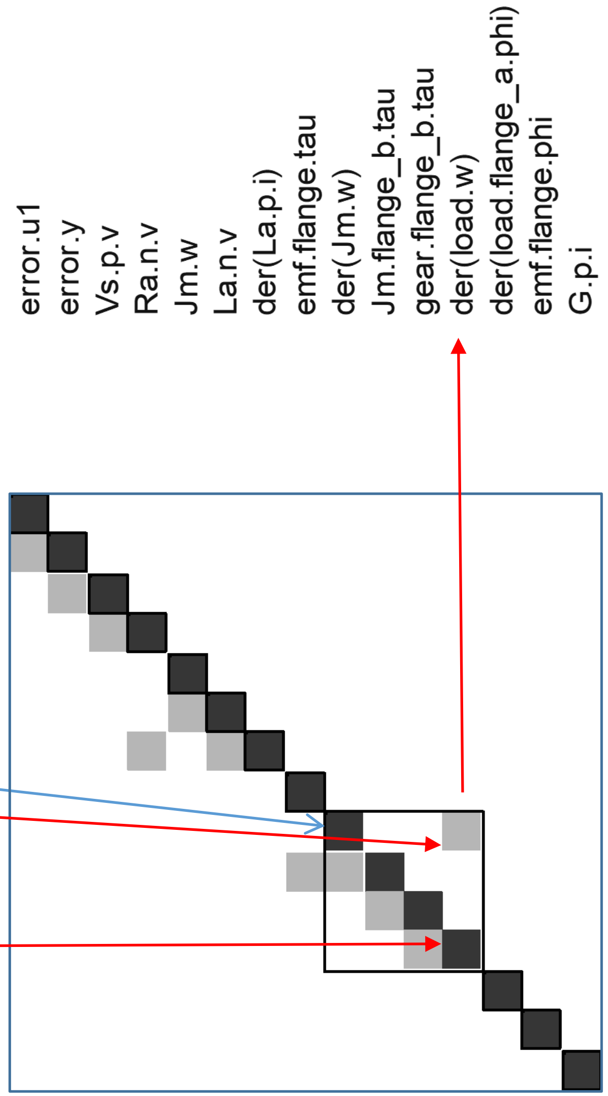
0 = **gear.flange_b.tau**-gear.ratio*Jm.flange_b.tau

load.J***der(load.w)** = -gear.flange_b.tau

der(load.flange_a.phi) = load.w

emf.flange.phi = gear.ratio*load.flange_a.phi

G.p.i+La.p.i = La.p.i



strongConnect (BLT)

```
"""
Find minimal systems of equations that have to be solved simultaneously.
Reference:
Tarjan, R. E. (1972), "Depth-first search and linear graph algorithms",
SIAM Journal on Computing 1 (2): 146–160, doi:10.1137/0201010
"""
function strongConnect(G, assign, v, nextnode, stack, components, lowlink, number)
const notOnStack = typemax(Int)

  if v == 0
    return nextnode
  end

  nextnode += 1
  lowlink[v] = number[v] = nextnode
  push!(stack, v)

  for w in [assign[j] for j in G[v]] # for w in the adjacency list of v
    if w > 0   # Is assigned
      if number[w] == 0 # if not yet numbered
        nextnode = strongConnect(G, assign, w, nextnode, stack, components, lowlink, number)
        lowlink[v] = min(lowlink[v], lowlink[w])
      else
        if number[w] < number[v]
        # (v, w) is a frond or cross-link
        # if w is on the stack of points. Always valid since otherwise number[w]=notOnStack (a big number)
          lowlink[v] = min(lowlink[v], number[w])
        end
      end
    end
  end

  if lowlink[v] == number[v]
    # v is the root of a component
    # start a new strongly connected component
    comp = []
    repeat = true
    while repeat
      # delete w from point stack and put w in the current component
      w = pop!(stack)
      number[w] = notOnStack
      push!(comp, w)
      repeat = w != v
    end
    push!(components, comp)
  end
  return nextnode
end
```

Julia AST for Meta-programming

- Quoted expression :( )
  - Any expression in LHS
- Operators are functions
- $ for "interpolation"

```
julia> equ = :(0 = x + 2y)
:(0 = x + 2y)

julia> dump(equ)
Expr
  head: Symbol =
  args: Array(Any,(2,))
    1: Int64 0
    2: Expr
      head: Symbol call
      args: Array(Any,(3,))
        1: Symbol +
        2: Symbol x
        3: Expr
          head: Symbol call
          args: Array(Any,(3,))
          typ: Any
      typ: Any
  typ: Any
```

```
julia> solved = Expr(:(=), equ.args[2].args[2], Expr(:call, :-, equ.args[2].args[3]))
:(x = -(2y))

julia> y = 10
10
julia> eval(solved)
-20
julia> @show x
x = -20

Julia> # Alternatively (interpolation by $):
julia> solved = :($(equ.args[2].args[2]) = - $(equ.args[2].args[3]))
```

Summary – *Modia* Prototype

- Modelica-like, but more powerful and simpler
- Algorithmic part: Julia functions (more powerful than Modelica functions)
- Model part: Julia meta-programming (no Modia parser)
- Equation part: Julia expressions (no Modia parser)
- Structural and Symbolic algorithms: Julia data structures / functions
- Target equations: Sparse DAE (no ODE)
- Simulation engine: IDA + KLU sparse matrix (Sundials 2.6.2)
- Revisiting all typically used algorithms: operating on arrays (no scalarization), improved algorithms for index reduction, overdetermined DAEs, switches, friction, Dirac impulses, ...
- Just-in-time compilation (build Modia model and simulate at once)

Next Immediate Steps

- Larger test suit
- Handle larger models (problem with code generation of big functions)
- Automated testing
- Coverage
- Julia package
- Proper web server (now Python SimpleJSONRPCServer and PyJulia)
- Cloud deployment
- Release to github (https://github.com/ModiaSim/Modia.jl)

- Begins on Saturday hackaton (hopefully with some help)

References

- Elmqvist H., Henningsson T. and Otter M. (2016): *System Modeling and Programming in a Unified Environment based on Julia*. Proceedings of ISoLA 2016 Conference Oct. 10-14, T. Margaria and B. Steffen (Eds.), Part II, LNCS 9953, pp. 198-217.
- Elmqvist H., Henningsson T., Otter M. (2017): *Innovations for future Modelica.* Modelica Conference 2017, Prague, May 15-17.
- Otter M., and Elmqvist H. (2017): *Transformation of Differential Algebraic Array Equations to Index One Form.* Modelica Conference 2017, Prague, May 15-17.