Introducing ArbFloats

powered by Fredrik Johansson's Arb, through William Hart's Nemo.jl

ArbFloats are intervals (midpoint ±radius)

values are viewed as floating point values: round(underlying, **n**)

n s.t. round(midpoint+radius, **n**) == round(midpoint-radius, **n**)

many functions are supported, including

iszero, ispositive, isinteger, isexact, midpoint, radius

ldexp, hypot, log, exp, (a)sin[h], (a)cos[h], (a)tan[h], atan2

floor, ceil, root, fib, gamma, lgamma, digamma, risingfactorial,

overlap, contains, zeta, agm

ArbFloats are best with data of narrow intervals

usually, the radius is within a factor of 1.5x..5x of best

as a guide, the radius may be within $1.5^{N_{ops}}$ of best

ArbFloat significands cover much ground

significand precision is settable to 7..1200 digits (24..4K bits)

on 64 bit machines, 35 digit significands use no indirect space

Amiable ArbFloats

```
using ArbFloats
setprecision(ArbFloat, 122)                           # 30 digits quite reliably

a = gamma(ArbFloat(33)); reciprocal_a = inv(a);
a, reciprocal_a
26313_083693_36935_30167_21801_21600_00000.0          # 1/4 trillion³
3.80039_075485_47435_92593_67089_27884_1279e-36       # 1/that

recovered_a = inv(reciprocal_a); showall(recovered_a)     # a with fuzz
2.63130_83693_36935_30167_21801_21600_00001e+35 ± 0.5634_9781_99377_656

a ≊ recovered_a
true

e = exp(ArbFloat(1));
bounds(e)
( 2.7182_8182_8459_0452_3536_0287_4713_5266_2 49 ,
  2.7182_8182_8459_0452_3536_0287_4713_5266_2 50 )

showsmart(e)
2.71828182845904523536028747135266 2 5_               # postfixes ~,+,-
                                                                            ...
```

Arb + BigFloat != ArbFloat

set bit precision to get well-behaved digits

digits	25	50	100	150	300	1000	digits
bits	110	175	355	520	1020	3345	bits

BigFloat and Arb

muladd	$5 \propto 8$	$2 \propto 3$	$3 \propto 4$	$1 \propto 1$	muladd
log	$1 \propto 8$	$1 \propto 7$	$1 \propto 7$	$3 \propto 4$	log
zeta	$1 \propto 9$	$1 \propto 75$	$1 \propto 48$	$1 \propto 32$	zeta
bits	125	250	500	4000	bits

BigFloat rounds values correctly, precise numbers that may or may not be known as accurate.
Arb is much faster for 150 digits and rounds to include the accurate value, less precisely.

ArbFloats are performant, mindful and honest.