Main results on distributions used in Dynare. We use the parametrization used in Distrubtions.jl

1 Inverse gamma

- AKA inverse gamma type 2
- shape: α
- scale: θ

$$
\begin{aligned}
\alpha &= \frac{\nu}{2} \\
\theta &= \frac{s}{2} \\
X \sim IG_2(\alpha, \theta) &\Leftrightarrow Z = X^{-1} \sim G(\alpha, \theta) \\
\mathbb{E}(X) &= \frac{\theta}{\alpha - 1} \\
\mathbb{V}ar(X) &= \frac{1}{\alpha - 2}[\mathbb{E}(X)]^2 \text{ for } \alpha > 2 \\
\alpha &= 2 + \frac{[\mathbb{E}(X)]^2}{\mathbb{V}ar(X)} \\
\theta &= (\alpha - 1)\mathbb{E}(X) \\
f_{IG_2}(x, \alpha, \theta) &= \frac{\theta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} e^{-\frac{\theta}{x}}
\end{aligned}
$$

2 Inverse gamma type I

$$
\begin{aligned}
X \sim IG_2(\alpha, \theta) &\Leftrightarrow Y = \sqrt{X} \sim IG_1(\alpha, \theta) \\
&\Leftrightarrow Z = X^{-1} \sim G(\alpha, \theta) \\
\mathbb{E}(Y) &= \sqrt{\theta}\frac{\Gamma(\alpha - \frac{1}{2})}{\Gamma(\alpha)} \text{ for } \alpha > \frac{1}{2} \\
\mathbb{V}ar(Y) &= \frac{\theta}{\alpha - 1} - [\mathbb{E}(Y)]^2 \text{ for } \alpha > 1 \\
\text{mode } Y &= \sqrt{\frac{\theta}{\alpha + \frac{1}{2}}} \\
f_{IG_1}(y, \alpha, \theta) &= f_{IG_2}(y^2, \alpha, \theta)|2y| \\
&= 2\frac{\theta^\alpha}{\Gamma(\alpha)} y^{-(2\alpha+1)} e^{-\frac{\theta}{y^2}}
\end{aligned}
$$

α solves

$$(\alpha-1)\left(\mathbb{V}ar(Y)+[\mathbb{E}(Y)]^2\right)-[\mathbb{E}(Y)]^2\,\frac{\Gamma(\alpha)}{\Gamma(\alpha-\frac{1}{2})}=0$$

and $\theta=(\alpha-1)(\mathbb{V}ar(Y)+[\mathbb{E}(Y)]^2)$

Appendices

Bauwens et (1999)

$$\begin{aligned}
X\sim IG_2(s,\nu) &\Leftrightarrow Y=\sqrt{X}\sim IG_1(s,\nu)\\
&\Leftrightarrow Z=X^{-1}\sim G(\frac{\nu}{2},\frac{2}{s})\\
\mathbb{E}(X) &= \frac{s}{\nu-2}\\
\mathbb{V}ar(X) &= \frac{2}{\nu-4}[\mathbb{E}(X)]^2 \text{ for } \nu>4\\
\mathbb{E}(Y) &= \sqrt{\frac{s}{2}}\frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \text{ for } \nu>1\\
\mathbb{V}ar(Y) &= \frac{s}{\nu-2}-[\mathbb{E}(Y)]^2 \text{ for } \nu>2\\
f_{IG_2}(x|\frac{\nu}{2},\frac{2}{s}) &= \frac{1}{\Gamma(\frac{\nu}{2})\left(\frac{2}{s}\right)^{\frac{\nu}{2}}}x^{-\frac{1}{2}(\nu+2)}e^{-\frac{s}{2x}}\\
f_{IG_2}(x|\alpha,\theta) &= \frac{1}{\Gamma(\alpha)\theta^\alpha}x^{-(\alpha+1)}e^{-\frac{1}{sx}}
\end{aligned}$$

Inverse gamma type I

Mode derivation

$$\begin{aligned}
\max_x y &= 2\frac{\theta^\alpha}{\Gamma(\alpha)}x^{-(2\alpha+1)}e^{-\frac{\theta}{x^2}}\\
\frac{dy}{dx} &= -\left[(2\alpha+1)x^{-1}-2\theta x^{-3}\right]y\\
&= 0\\
x^\star &= \sqrt{\frac{\theta}{\alpha+\frac{1}{2}}}
\end{aligned}$$

Obtaining α and θ from mean and variance

$$\begin{aligned}
\theta &= [\mathbb{E}(Y)]^2\left(\frac{\Gamma(\alpha)}{\Gamma(\alpha-\frac{1}{2})}\right)^2\\
\theta &= (\alpha-1)\left(\mathbb{V}ar(Y)+[\mathbb{E}(Y)]^2\right)
\end{aligned}$$

Solve numerically

$$(\alpha - 1)\left(\mathbb{V}ar(Y) + [\mathbb{E}(Y)]^2\right) - [\mathbb{E}(Y)]^2 \left(\frac{\Gamma(\alpha)}{\Gamma(\alpha - \frac{1}{2})}\right)^2 = 0$$

or taking the logarithm for numerical stabilitye:

$$\ln(\alpha - 1) + \ln\left(\mathbb{V}ar(Y) + [\mathbb{E}(Y)]^2\right) - \ln [\mathbb{E}(Y)]^2 - 2 * \ln\left(\frac{\Gamma(\alpha)}{\Gamma(\alpha - \frac{1}{2})}\right) = 0$$